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Erratum: Note on integrability of marginally deformed ABJ(M) theories

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Some computation errors were found. The conclusion about integrability is still valid. Eq. (3.5) should be changed to

$$\begin{aligned} \left(\widetilde{\mathbb{P}}_{i,i+2} \right)_{J_i J_{i+1} J_{i+2}}^{I_i I_{i+1} I_{i+2}} &\equiv \exp \left(-2\pi i \gamma \left(Q^{J_i} \times Q^{J_{i+1}} + Q^{J_{i+1}} \times Q^{J_{i+2}} + Q^{J_{i+2}} \times Q^{J_i} \right) \right) \\ &\times \left(\mathbb{P}_{i,i+2} \right)_{J_i J_{i+1} J_{i+2}}^{I_i I_{i+1} I_{i+2}}. \end{aligned} \quad (3.5)$$

Eqs. (4.1)–(4.3) should be changed to

$$\widetilde{\mathfrak{R}}^{44}(u)_{KL}^{IJ} = \exp \left(i\pi \gamma \left(Q^J \times Q^I - Q^K \times Q^L \right) \right) \mathfrak{R}^{44}(u)_{KL}^{IJ}, \quad (4.1)$$

$$\widetilde{\mathfrak{R}}^{4\bar{4}}(u)_{KL}^{IJ} = \exp \left(-i\pi \gamma \left(Q^J \times Q^I - Q^K \times Q^L \right) \right) \mathfrak{R}^{4\bar{4}}(u)_{KL}^{IJ}, \quad (4.2)$$

$$\widetilde{\mathfrak{R}}^{\bar{4}4}(u)_{KL}^{IJ} = \exp \left(-i\pi \gamma \left(Q^J \times Q^I - Q^K \times Q^L \right) \right) \mathfrak{R}^{\bar{4}4}(u)_{KL}^{IJ}, \quad (4.3)$$

$$\widetilde{\mathfrak{R}}^{\bar{4}\bar{4}}(u)_{KL}^{IJ} = \exp \left(i\pi \gamma \left(Q^J \times Q^I - Q^K \times Q^L \right) \right) \mathfrak{R}^{\bar{4}\bar{4}}(u)_{KL}^{IJ}, \quad (4.4)$$

Eqs. (4.9)–(4.12) should be changed to

$$\widetilde{\mathfrak{R}}^{44}(u)_{KL}^{IJ} = u \exp \left(2\pi i \gamma Q^J \times Q^I \right) \delta_K^I \delta_L^J + \delta_L^I \delta_K^J, \quad (4.9)$$

$$\widetilde{\mathfrak{R}}^{4\bar{4}}(u)_{KL}^{IJ} = -(u+2) \exp \left(-2\pi i \gamma Q^J \times Q^I \right) \delta_K^I \delta_L^J + \delta^{IJ} \delta_{KL}, \quad (4.10)$$

$$\widetilde{\mathfrak{R}}^{44}(u)_{KL}^{IJ} = -(u+2) \exp(-2\pi i \gamma Q^J \times Q^I) \delta_K^I \delta_L^J + \delta^{IJ} \delta_{KL}, \quad (4.11)$$

$$\widetilde{\mathfrak{R}}^{44}(u)_{KL}^{IJ} = u \exp(2\pi i \gamma Q^J \times Q^I) \delta_K^I \delta_L^J + \delta_L^I \delta_K^J. \quad (4.12)$$

Eq. (5.1) should be changed to

$$\begin{aligned} \tilde{\Lambda}(\nu) = & 2^{-L} (\nu+1)^L (-\nu-2)^L \exp\left(-\frac{i}{2}\pi\gamma L - \frac{i}{2}\gamma N_m + i\pi\gamma N_r\right) \prod_{a=1}^{N_l} \frac{\nu + il_a - \frac{1}{2}}{\nu + il_a + \frac{1}{2}} \\ & + 2^{-L} (\nu+1)^L (-\nu)^L \exp\left(-\frac{i}{2}\pi\gamma L - \frac{i}{2}\gamma N_m + i\pi\gamma N_l\right) \prod_{c=1}^{N_r} \frac{\nu + ir_c + \frac{5}{2}}{\nu + ir_c + \frac{3}{2}} \\ & + 2^{-L} (-\nu-2)^L \nu^L \exp\left(\frac{i}{2}\pi\gamma L + \frac{i}{2}\pi\gamma N_m - i\pi\gamma N_r\right) \prod_{a=1}^{N_l} \frac{\nu + il_a + \frac{3}{2}}{\nu + il_a + \frac{1}{2}} \prod_{b=1}^{N_m} \frac{\nu + im_b}{\nu + im_b + 1} \\ & + 2^{-L} (-\nu-2)^L \nu^L \exp\left(\frac{i}{2}\pi\gamma L + \frac{i}{2}\pi\gamma N_m - i\pi\gamma N_l\right) \prod_{c=1}^{N_r} \frac{\nu + ir_c + \frac{1}{2}}{\nu + ir_c + \frac{3}{2}} \prod_{b=1}^{N_m} \frac{\nu + im_b + 2}{\nu + im_b + 1}. \end{aligned} \quad (5.1)$$

Eqs. (5.3)–(5.6) should be changed to

$$\begin{aligned} \tilde{\Lambda}(\nu) = & 2^{-L} (-\nu)^L (\nu+1)^L \exp\left(\frac{i}{2}\pi\gamma L + \frac{i}{2}\pi\gamma N_m - i\pi\gamma N_r\right) \prod_{a=1}^{N_l} \frac{\nu + il_a + \frac{5}{2}}{\nu + il_a + \frac{3}{2}} \\ & + 2^{-L} (\nu+1)^L (-\nu-2)^L \exp\left(\frac{i}{2}\pi\gamma L + \frac{i}{2}\pi\gamma N_m - i\pi\gamma N_l\right) \prod_{c=1}^{N_r} \frac{\nu + ir_c - \frac{1}{2}}{\nu + ir_c + \frac{1}{2}} \\ & + 2^{-L} (-\nu-2)^L \nu^L \exp\left(-\frac{i}{2}\pi\gamma L - \frac{i}{2}\pi\gamma N_m + i\pi\gamma N_r\right) \prod_{a=1}^{N_l} \frac{\nu + il_a + \frac{1}{2}}{\nu + il_a + \frac{3}{2}} \prod_{b=1}^{N_m} \frac{\nu + im_b + 2}{\nu + im_b + 1} \\ & + 2^{-L} (-\nu-2)^L \nu^L \exp\left(-\frac{i}{2}\pi\gamma L - \frac{i}{2}\pi\gamma N_m + i\pi\gamma N_l\right) \prod_{b=1}^{N_m} \frac{\nu + im_b}{\nu + im_b + 1} \prod_{c=1}^{N_r} \frac{\nu + ir_c + \frac{3}{2}}{\nu + ir_c + \frac{1}{2}}. \\ & \exp(i\pi\gamma L + i\pi\gamma N_m - 2\pi i \gamma N_l) \left(\frac{l_a - \frac{i}{2}}{l_a + \frac{i}{2}}\right)^L = \prod_{a' \neq a} \frac{l_a - l_{a'} - i}{l_a - l_{a'} + i} \prod_{b=1}^{N_m} \frac{l_a - m_b + \frac{i}{2}}{l_a - m_b - \frac{i}{2}}, \\ & \exp(-i\pi\gamma L - i\pi\gamma N_m + 2\pi i \gamma N_l) \left(\frac{r_c - \frac{i}{2}}{r_c + \frac{i}{2}}\right)^L = \prod_{b=1}^{N_m} \frac{r_c - m_b + \frac{i}{2}}{r_c - m_b - \frac{i}{2}} \prod_{c' \neq c} \frac{r_c - r_{c'} - i}{r_c - r_{c'} + i}, \\ & \exp(-i\pi\gamma N_l + i\pi\gamma N_r) = \prod_{a=1}^{N_l} \frac{m_b - l_a - \frac{i}{2}}{m_b - l_a + \frac{i}{2}} \prod_{b \neq b'} \frac{m_b - m_{b'} - i}{m_b - m_{b'} + i} \prod_{c=1}^{N_r} \frac{m_b - r_c + \frac{i}{2}}{m_b - r_c - \frac{i}{2}}. \end{aligned} \quad (5.3)$$

$$P_{total} = \frac{1}{i} \left[\log \tilde{\Lambda}(0) + \log \tilde{\Lambda}(0) \right]$$

$$= \frac{1}{i} \left[i\pi\gamma N_r - i\pi\gamma N_l + \sum_{a=1}^{N_l} \log \frac{il_a - \frac{1}{2}}{il_a + \frac{1}{2}} + \sum_{c=1}^{N_r} \log \frac{ir_c - \frac{1}{2}}{ir_c + \frac{1}{2}} \right]. \quad (5.5)$$

$$1 = \exp(i\pi\gamma N_r - i\pi\gamma N_l) \prod_{a=1}^{N_l} \frac{il_a - \frac{1}{2}}{il_a + \frac{1}{2}} \prod_{c=1}^{N_r} \frac{ir_c - \frac{1}{2}}{ir_c + \frac{1}{2}}. \quad (5.6)$$

Eq. (5.7), the frist line, $3L$ should be added after $=$.

Eqs. (A.2)–(A.3) should be changed to

$$\begin{aligned}
 & 2^{-L} (\mathbb{P}_{01})_{K_1 J_1}^{K_0 I_1} \widetilde{\mathfrak{R}}^{44}_{02}(a)^{K_1 I_2} \cdots \widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{K_{2i-3} I_{2i-2}} \delta_{K_{2i-2} J_{2i-2}}^{K_{2i-2}} e^{-2\pi i \gamma Q^{K_{2i-1} \times Q^{I_{2i-1}}}} \\
 & \delta_{J_{2i-1}}^{I_{2i-1}} \widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{K_{2i-1} I_{2i}} \cdots (\mathbb{P}_{0(2L-1)})_{K_{2L-1} J_{2L-1}}^{K_{2L-2} I_{2L-1}} \widetilde{\mathfrak{R}}^{44}_{0(2L)}(a)^{K_{2L-1} I_{2L}} \\
 & = 2^{-L} \widetilde{\mathfrak{R}}^{44}_{02}(a)^{I_1 I_2}_{J_3 J_2} \cdots \widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{I_{2i-3} I_{2i-2}}_{K_{2i-1} J_{2i-2}} e^{-2\pi i \gamma Q^{K_{2i-1} \times Q^{I_{2i-1}}}} \delta_{J_{2i-1}}^{I_{2i-1}} \widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{K_{2i-1} I_{2i}}_{J_{2i+1} J_{2i}} \\
 & \quad \cdots \widetilde{\mathfrak{R}}^{44}_{0(2L)}(a)^{I_{2L-1} I_{2L}}_{J_1 J_{2L}}. \tag{A.2}
 \end{aligned}$$

$$\begin{aligned}
 & 2^{-L} (\mathbb{P}_{01})_{K_1 J_1}^{K_0 I_1} \widetilde{\mathfrak{R}}^{44}_{02}(a)^{K_1 I_2} \cdots \widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{K_{2i-3} I_{2i-2}} (\mathbb{P}_{0(2i-1)})_{K_{2i-1} J_{2i-1}}^{K_{2i-2} I_{2i-1}} (-\mathbb{I})_{K_{2i-1} J_{2i}}^{K_{2i-1} I_{2i}} \\
 & e^{2\pi i \gamma Q^{K_{2i} \times Q^{I_{2i}}}} (\mathbb{P}_{0(2i+1)})_{K_{2i+1} J_{2i+1}}^{K_{2i} I_{2i+1}} \widetilde{\mathfrak{R}}^{44}_{0(2i+2)}(a)^{K_{2i+1} I_{2i+2}} \cdots \widetilde{\mathfrak{R}}^{44}_{02L}(a)^{K_{2L-1} I_{2L}}_{K_0 J_{2L}} \\
 & = -2^{-L} \widetilde{\mathfrak{R}}^{44}_{02}(a)^{I_1 I_2}_{J_3 J_2} \cdots \widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{I_{2i-3} I_{2i-2}}_{J_{2i-1} J_{2i-2}} \delta_{J_{2i}}^{I_{2i}} \delta_{J_{2i+1}}^{I_{2i-1}} e^{2\pi i \gamma Q^{I_{2i-1} \times Q^{I_{2i}}}} \\
 & \quad \widetilde{\mathfrak{R}}^{44}_{0(2i+2)}(a)^{I_{2i+1} I_{2i+2}}_{J_{2i+3} J_{2i+2}} \cdots \widetilde{\mathfrak{R}}^{44}_{0(2L)}(a)^{I_{2L-1} I_{2L}}_{J_1 J_{2L}}. \tag{A.3}
 \end{aligned}$$

Eqs. (A.4)–(A.6) should be changed to

$$\tilde{\tau}^{-1}(0, a) = 2^L \left[\widetilde{\mathfrak{R}}^{44}_{0(2L)}(a)^{-1} \right]_{I_{2L-1} I_{2L}}^{J_1 J_{2L}} \cdots \left[\widetilde{\mathfrak{R}}^{44}_{0(2)}(a)^{-1} \right]_{I_1 I_2}^{J_3 J_2}, \tag{A.4}$$

$$\left[\widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{-1} \right]_{I_{2i+1} I_{2i}}^{J_{2i-1} J_{2i}} = - \left(\frac{1}{a+2} \mathbb{I}_{I_{2i+1} I_{2i}}^{J_{2i-1} J_{2i}} e^{-2\pi i \gamma Q^{J_{2i-1} \times Q^{J_{2i}}}} + \frac{1}{a^2 - 4} \mathbb{K}_{I_{2i+1} I_{2i}}^{J_{2i-1} J_{2i}} \right). \tag{A.5}$$

$$\begin{aligned}
 & \tilde{\tau}^{-1}(u, a) \tilde{\tau}'(u, a)|_{u=0} \\
 & = \sum_{i=1}^L \mathbb{I}_{J_1 \cdots J_{2i-3}}^{K_1 \cdots K_{2i-3}} \otimes \left[\widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{-1} \right]_{I_{2i-1} I_{2i}}^{K_{2i+1} K_{2i}} \left[\widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{-1} \right]_{I_{2i-3} I_{2i-2}}^{K_{2i-1} K_{2i-2}} \\
 & \quad e^{-2\pi i \gamma Q^{\tilde{K}_{2i-1} \times Q^{I_{2i-1}}}} \delta_{J_{2i-1}}^{I_{2i-1}} \widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{I_{2i-3} I_{2i-2}}_{\tilde{K}_{2i-1} J_{2i-2}} \widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{\tilde{K}_{2i-1} I_{2i}}_{J_{2i+1} J_{2i}} \otimes \mathbb{I}_{J_{2i+2} \cdots J_{2L}}^{K_{2i+2} \cdots K_{2L}} \\
 & \quad + \sum_{i=1}^L \mathbb{I}_{J_1 \cdots J_{2i-1}}^{K_1 \cdots K_{2i-1}} \otimes \left[\widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{-1} \right]_{I_{2i-1} I_{2i}}^{K_{2i+1} K_{2i}} \left(-\delta_{J_{2i+1}}^{I_{2i-1}} \delta_{J_{2i}}^{I_{2i}} \right) e^{2\pi i \gamma Q^{I_{2i+1} \times Q^{I_{2i}}}} \otimes \mathbb{I}_{J_{2i+2} \cdots J_{2L}}^{K_{2i+2} \cdots K_{2L}} \\
 & = \sum_{i=1}^L \mathbb{I}_{J_1 \cdots J_{2i-3}}^{K_1 \cdots K_{2i-3}} \otimes \left[\widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{-1} \right]_{J_{2i-1} I_{2i}}^{K_{2i+1} K_{2i}} \left[\widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{-1} \right]_{I_{2i-3} I_{2i-2}}^{K_{2i-1} K_{2i-2}} e^{-2\pi i \gamma Q^{\tilde{K}_{2i-1} \times Q^{J_{2i-1}}}} \\
 & \quad \widetilde{\mathfrak{R}}^{44}_{0(2i-2)}(a)^{I_{2i-3} I_{2i-2}}_{\tilde{K}_{2i-1} J_{2i-2}} \widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{\tilde{K}_{2i-1} I_{2i}}_{J_{2i+1} J_{2i}} \otimes \mathbb{I}_{J_{2i+2} \cdots J_{2L}}^{K_{2i+2} \cdots K_{2L}} \\
 & \quad - \sum_{i=1}^L \mathbb{I}_{J_1 \cdots J_{2i-1}}^{K_1 \cdots K_{2i-1}} \otimes \left[\widetilde{\mathfrak{R}}^{44}_{0(2i)}(a)^{-1} \right]_{J_{2i+1} J_{2i}}^{K_{2i+1} K_{2i}} e^{2\pi i \gamma Q^{J_{2i+1} \times Q^{J_{2i}}}} \otimes \mathbb{I}_{J_{2i+2} \cdots J_{2L}}^{K_{2i+2} \cdots K_{2L}} \\
 & = \sum_{i=1}^L \frac{1}{a^2 - 4} ((a-2)\mathbb{I} + (a^2 - 4)\tilde{\mathbb{P}}_{2i-1, 2i+1} - (a-2)\mathbb{P}_{2i-1, 2i+1} \mathbb{K}_{2i-1, 2i} \\
 & \quad + (a+2)\mathbb{P}_{2i-1, 2i+1} \mathbb{K}_{2i, 2i+1}) \tag{A.6}
 \end{aligned}$$

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